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STIMULATING SLIPPAGE IN BALL BEARINGS BY INVESTIGATING THE CRITICAL ANGLE OF SLIPPAGE OF A SOLID SPHERE

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ABSTRACT

Dissipative losses in ball bearings occur due to slippage in machine parts. This study examines how the slippage of a solid marble varies with its angle of inclination. During slippage, experimental linear velocity at the bottom of the ramp is greater than theory predicts. The ratio of translational/rotational KE, which theory predicts as 2.5, increases beyond $\theta_{c,}$ reaching (15.86 ± 0.68) for 75°

Photogates were placed at bottom of incline of length (0.575 ± 0.001) m. The linear velocity of a solid marble with mass (19.46 ± 0.001) g was measured in 8 trials for angles between 5-75°; μ_s between the marble and an acrylic incline was measured to be 0.264 ± 0.002. The critical angle, $\tan^{-1}(7\mu_s/2)$, was estimated at 42.7°± 0.3°.

The experimental velocity exceeded theory beyond error at 30°, invalidating our prediction of 42.7°. While the values for θ = 5° was accurate to 0.01 m/s, θ = 75° differed by 0.41 m/s. The ratio E_k/E_r was 2.5 only till 30°. The angular velocity peaked at 30° and rapidly fell beyond, because of slippage. To prove lack of friction caused slippage, the marble was replaced with rubber ball with greater μ_s. Slippage occurred only beyond 60°.

Keywords: *Slippage, ball bearings, friction, energy dissipation.*

I. INTRODUCTION

Energy dissipation during rotational motion is a complex phenomenon and one with several practical applications, especially in improving the working efficiency of rolling parts of machines and carts. Slippage, in particular, is a form of inefficiency in machine parts, and this research paper is motivated by the desire to investigate the specific angles of operation needed to minimize slipping, leading to the research question:

"What is the critical angle of an inclined plane beyond which a sphere undergoing rotational motion on the plane will begin to slip?"
By determining the angles of operation that allows for pure rolling, machines and industrial gears can be optimised to as to not operate beyond the critical angle, hence reducing mechanical inefficiency by reducing energy dissipation.

The investigation includes a consideration of Newton's laws and rotational dynamics to hypothesize a value for the critical angle; this is then verified experimentally. By recording the marble's linear velocity as it rolls using a photogate, we then evaluate our theory based on two objectives: firstly, at what angle does slippage begin to occur, and secondly, whether the extent of slippage increases with increasing angle. The essay concludes with a critical evaluation of our results and explanations for the disparities between theory and experiment, suggesting improvements for the experiment to increase the strength of our conclusion.

II. METHODS & MATERIALS

Background Literature
Forces which act on bodies can produce two forms of motion: translatory and rotational. If the net force acting on a body is directed towards its centre of mass, the body undergoes translatory motion, where the centre of mass of a

body is displaced relative to a fixed point. Forces not directed at the body's centre of mass produce a torque, causing rotation about a fixed axis. Antiparallel forces tangent to the surface of the body and acting on diametrically opposite points generate a couple, causing rotational motion and no translation. If only one of these forces is present, the body translates as well as rotates.

The lattermost case is applicable to a sphere rolling down an incline:

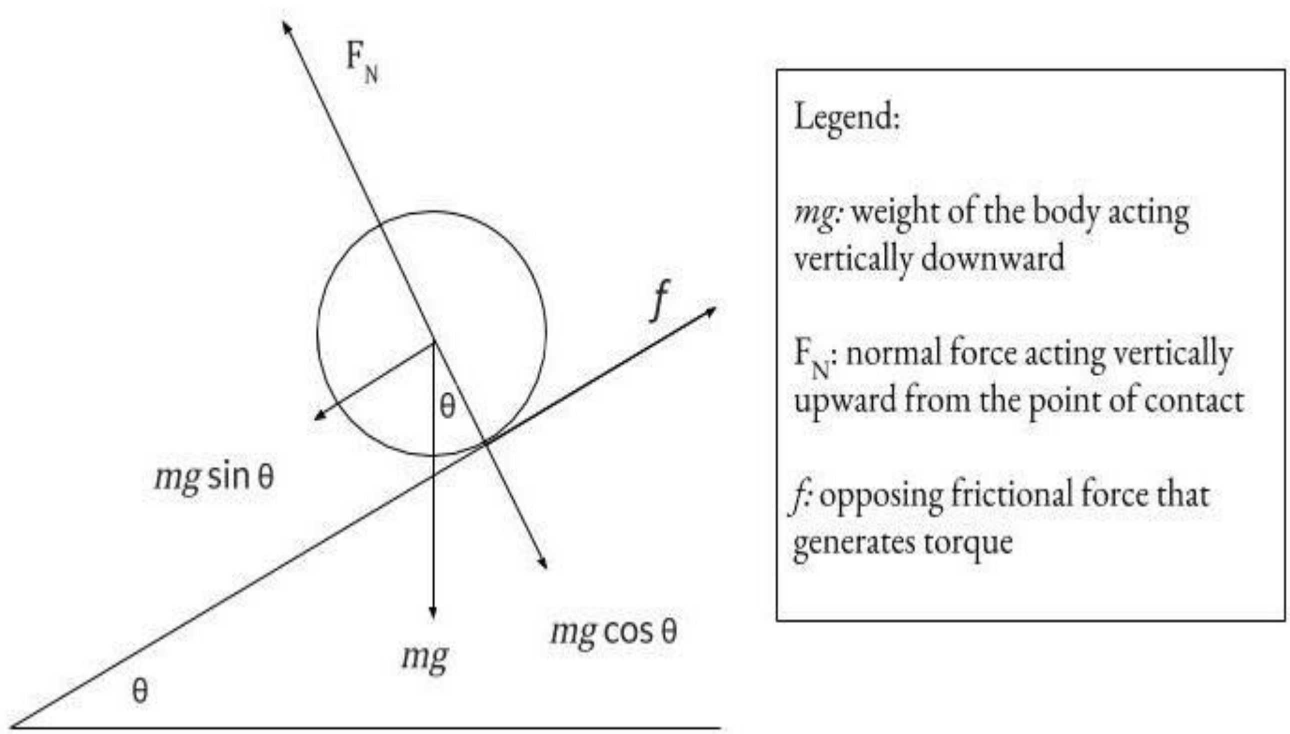

Figure 1: Free-body diagram for a sphere on an incline

The sphere not only accelerates down the incline, but undergoes rotation about an axis normal to the surface to the sphere and parallel to the incline. The force of friction originating from the point of contact of the sphere and incline generates the torque responsible for rolling. When the sphere's centre of mass travels a distance equal to the circumference of the body in one rotation, it is said to experience pure rolling. In other words, the point of contact on the sphere is instantaneously at rest relative to the contact surface.

In contrast to pure translatory motion, the work done against friction is not dissipative during pure rotation but is rather converted to rotational kinetic energy. Further, conservation laws dictate that the sphere's gravitational potential energy is converted to translatory kinetic energy and rotational kinetic energy in a fixed ratio of 5:2 as the sphere rolls. Under this condition of no slippage, $v = r\omega$ holds, where v= linear velocity of the sphere, $\omega = \frac{d\theta}{dt}$= angular velocity of the sphere about its axis of rotation, and $r$is the radius of the sphere. .

Static friction, $\mu_s mgcos\theta,$ is proportional to cos θ, which is strictly decreasing across the domain [0°, 90°]. We predict that there is a critical angle θ_c beyond which the torque generated will be insufficient to produce pure rolling, and the sphere will begin to slip, where $v_{CoM} > 2\pi r$. Further, the ratio of the body's translational KE/rotational KE will be greater than 2.5, implying that the body's velocity at the bottom of the incline will be greater than predicted by theory. This observation forms the basis for the experimental design.

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Experimental Design

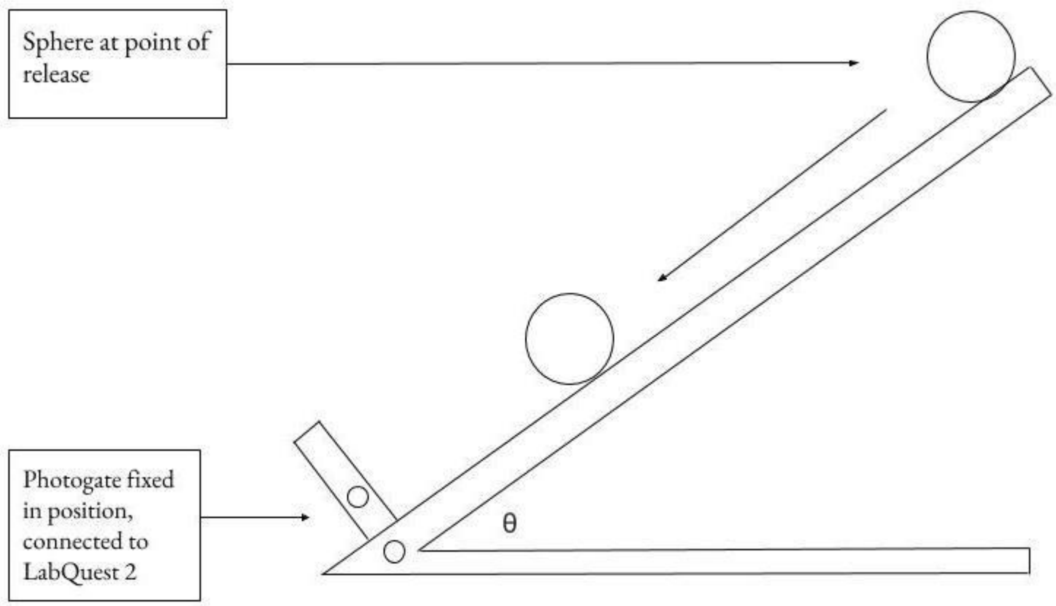


Figure 2: Schematic Layout of Apparatus

The objective of the experiment is to measure the linear velocity of a sphere in rotational motion. A spherical marble of radius (1.23 ± 0.005) cm and mass (19.46 ± 0.01) g is rolled from the exact top of a smooth, acrylic-plated incline θ. A photogate with precision ± 10^{-5} s, is placed such that its beam is at a distance (57.5 ± 0.1) cm from the point the marble is released. The angle of incline θ is measured by a protractor to ± 0.5^{o}; the photogate is connected to Vernier LabQuest 2 software and graphing software to collect raw data.

Figure 3: Marble used in experiment. 1 cm on document = 0.49 cm real size

The aforementioned set-up was chosen instead of a motion sensor as the latter recorded inaccurate data due to signals that reflected from the marble to the incline, causing interference. Further, the motion sensor failed to calibrate and produced excessive background noise.

Photogate Description and Instrument Corrections

When a body passes through a photogate, it momentarily cuts across a laser beam. By measuring the amount of time the beam is cut and the length of the sector it cuts through, we can measure the ball's instantaneous velocity: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \approx \frac{dx}{dt}$

It is important to note that we cannot take Δx as the marble's diameter because the laser beam does not cut through the equator, but rather a plane parallel to the equator. By measuring the distance using a calliper (after correcting for zero error) from the base of the photogate to the laser source $b$as (0.90 ± 0.005) cm, we can apply Pythagoras' Theorem to find Δx, where $\Delta x = 2a$ (view diagram below). Letting r = (1.23 ± 0.005) cm and propagating uncertainties, we get a = (1.18 ± 0.01) cm. Thus, Δx= (2.37 ± 0.03) cm. Given that the uncertainty in $t \approx \mathbf{0}$, our experimental velocity can be recorded as $v = (\frac{2.37}{\Delta t}$± 1.1%) m s^{-1}.

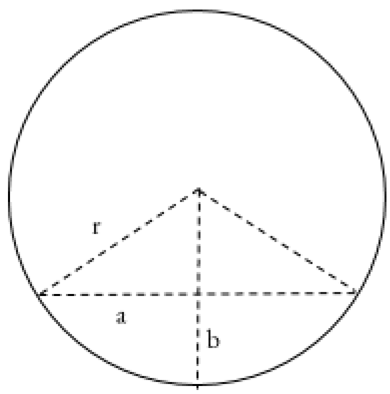


Figure 4: Schema of Marble

$$a^2 = r^2 - (r-b)^2$$

III. THEORETICAL FRAMEWORK AND HYPOTHESIS

This section builds a theoretical model to compare to the experimental results. Derivations of the critical slippage angle, linear velocity on the incline and E_t/E_r are provided, along with an experimental determination of the coefficient of static friction between the marble and the incline, enabling us to hypothesize a numerical value for the critical slippage angle.

a) Determining the Critical Angle for Slippage

We earlier defined the angle of slippage θ_c such that the relation $v = r\omega$ just holds.
If the sphere were to slip, it would rotate less around its axis and translate more, or $v \geq r\omega$. Differentiating with respect to time, we get $a \geq r\alpha$, where $\alpha$is the marble's angular acceleration about its axis of rotation.
The following two equations (Verma, 183) describe the motion of the marble as it descends down the ramp:

$$mgsin\theta - f = ma \qquad \text{......(1)}$$
$$f \times r = \Gamma = I\alpha \qquad \text{......(2)}$$

Where Γrepresents the net torque on the body, and Irepresents the moment of inertia of the sphere about the axis of rotation.

$$I = \sum_i m_i r_i^2 = \int r^2 dm = \frac{2}{5}mr^2\,(for\ a\ solid\ sphere) \qquad \text{(Verma, 178)}$$

Using the above, (ii) becomes:

$$fr = (\frac{2}{5}mr^2)(\frac{a}{r}) \qquad \text{......(3)}$$

$$f = \frac{2}{5}ma \qquad \text{......(4)}$$

Combining (i) and (iv)

$$a = \frac{5}{7}gsin\theta \qquad \text{......(5)}$$

$$f = \frac{2}{7}mgsin\theta \qquad \text{......(6)}$$

Under the limiting condition of friction producing pure rolling, we have:

$$\mu_s mgcos\theta \geq \frac{2}{7}\ mgsin\ \theta \qquad \text{......(7)}$$

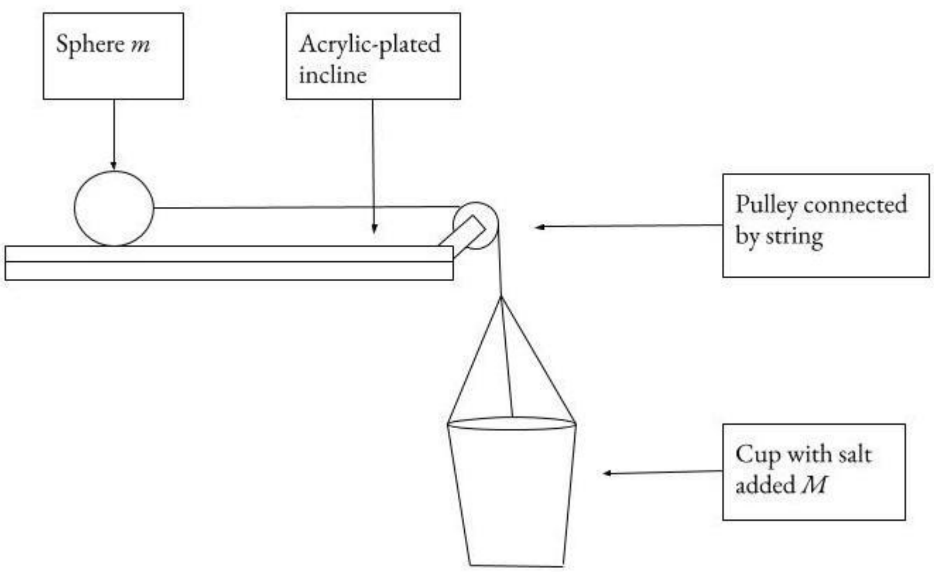
This simplifies to:

$$\theta_c = arctan\ (\frac{7}{2}\mu_s) \qquad \text{......(8)}$$

These results enable us to predict when the marble will begin to slip, based on μ_s. For all angles < θ_c the force of friction generates the torque needed for rotation and so no energy is dissipated; beyond this angle, energy is dissipated in work done against friction in the form of heat and sound.

b) Determining the Coefficient of Static Friction

To use (8) to find a numerical value for θ_c, we used a simple laboratory set-up to measure μ_s between the marble and the incline surface.



The marble is placed on the incline is tied to a string, which is connected via a pulley to a styrofoam cup. Salt is continually added to the cup till it reaches a mass Min small increments of approximately 0.1 g until the marble just starts to slide when we tap the incline plane. The tension in the rope, $Mg$is equal to the opposing force of friction, $\mu_s mg$where $m$is the mass of the marble. The coefficient of static friction μ_s is then equal to $\frac{M}{m}$

During the experiment, it was noticed that the string was also being dragged along the incline plane, which would provide a false reading for μ_s . To correct this error, the plane was inclined to $(1\pm0.5)^o$, so that the string would not be in contact with the surface. This introduces a factor of cos 1^o in the relation for μ_s , but using the small angle approximation of cos x ≈1 for small x, $\mu_s \approx \frac{M}{m}$.

6 trials were conducted, with the cup being emptied and refilled with salt each time.

Given m = (19.46 ± 0.01) g, $\theta_{incline} = (1\pm0.5)^o$

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Table 1: Coefficient of static friction between the incline plane and marble

Trial No.	Mass of Cup M in g (± 0.01 g)	Coefficient of Static Friction μ_s
1	5.13	0.264
2	5.17	0.266
3	5.17	0.266
4	5.09	0.262
5	5.11	0.263
6	5.12	0.263
Mean	**5.13 ± 0.04**	**0.264 ± 0.002**

Substituting this value of μ_s in (ix), we arrive at an answer of $(42.7 \pm 0.3)^o$ for θ_c

c) Determining the Linear Velocity and E_t/E_r Ratio

The next part of our framework is determining the velocity, in theory, that the ball is expected to have as it crosses the photogate, as well as the expected E_t/E_r ratio. We apply the principle of energy conservation:

$$E_p = E_k + E_r \Rightarrow mgh = \frac{1}{2}(mv^2 + I\omega^2) \qquad \text{......(9)}$$

Substituting I and $h = dsin\theta$ and from the relation derived earlier,

$$mgdsin\theta = \frac{1}{2}(mv^2 + \frac{2}{5}mr^2w^2) \qquad \text{......(10)}$$

Using the relation $v = \omega r$, which holds under the condition of pure rolling, we get

$$mgdsin\theta = \frac{7}{10}mv^2 \qquad \text{......(11)}$$

Cancelling m and rearranging,

$$v = \sqrt{\frac{10}{7}gdsin\theta} \qquad \text{......(12)}$$

Now that we have a relation for our expected linear velocity, we can theorize our expected E_t/E_r ratio.
Notice that our total energy is equal to 7/10th of the product of the marble's mass and its linear velocity squared (equation 11). Since translational kinetic energy is equal to $\frac{1}{2}mv^2$ our rotational kinetic energy is equal to $mv^2(\frac{7}{10} - \frac{1}{2}) = \frac{2}{10}mv^2$. E_t/E_r is thus $\frac{1}{2}mv^2/\frac{2}{10}mv^2 = \frac{10}{4} = 2.5$

Before moving on, however, it is interesting to note that we could have considered the marble's motion down the plane to be purely rotational, if we let the axis of rotation be such that it passes through the point of contact of the marble and incline and parallel to the incline. Then, using the Parallel Axes Theorem, the modified moment of inertia would have been $I_M = \frac{7}{5}mr^2$. Since the marble's kinetic energy would be $\frac{1}{2}I_M\omega^2$ with no translatory energy, this would have yielded the same results as in (12).

We now have all the theoretical information we need to verify our experimental set-up.

IV. RESULTS AND DISCUSSION

8 different trials were conducted for each angle from a range of angles between 5º-75º in increments of 5º. Readings were taken only till 75º as recording accurate readings beyond this angle was unfeasible. only those readings were accepted where the marble passed directly through the photogate, without colliding with the boundaries of the incline or the photogate itself. The following table details the mean values for the 8 trials. The full set of data is present in Appendix I.

Table 2: experimental vs theoretical velocity of the marble for a range of angles

Angle of Incline θ (±0.5)º	Mean Beam Cutting Time (± 10^{-5} s)	Experimental Velocity (ms^{-1})(± 1.1 %)	Theoretical Velocity (ms^{-1}) ≈(± 0 %)
5	0.02848	0.83	0.84
10	0.02007	1.18	1.18
15	0.01642	1.44	1.44
20	0.01437	1.65	1.66
25	0.01289	1.84	1.84
30	0.01154	2.05	2.01
35	0.01061	2.23	2.15
40	0.00985	2.40	2.27
45	0.00928	2.55	2.39
50	0.00882	2.69	2.48
55	0.00848	2.79	2.57
60	0.00813	2.93	2.64
65	0.00787	3.01	2.70
70	0.00763	3.11	2.75
75	0.00741	3.20	2.79

A number of interesting observations can be made from the above sample of processed data: the first is the remarkable precision with which the experimental values align with what the theory predicted for angles in the range 5º-25º, with a maximum error of ± 0.01 m s^{-1}. The lack of systematic errors arises from a relatively simplistic set-up and steps taken to mitigate the influence of external factors: after each trial, the marble was cleaned and the photogate was recalibrated. Further, even within the 8 trials per reading for Mean Beam Cutting Time, there was minimum variation in their values [view Appendix I] , implying that random errors had also been minimized. The readings of the two velocities begin to differ in the 30º-35º range, and the difference goes on increasing as the angle increases, reaching 0.41 m s^{-1} for the final reading. When contrasting the change in velocity of the marble between

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different angles, we notice that the difference decreases with increasing values of θ, supporting the theory that the velocity with increasing θ of a ball follows a square-root relationship.

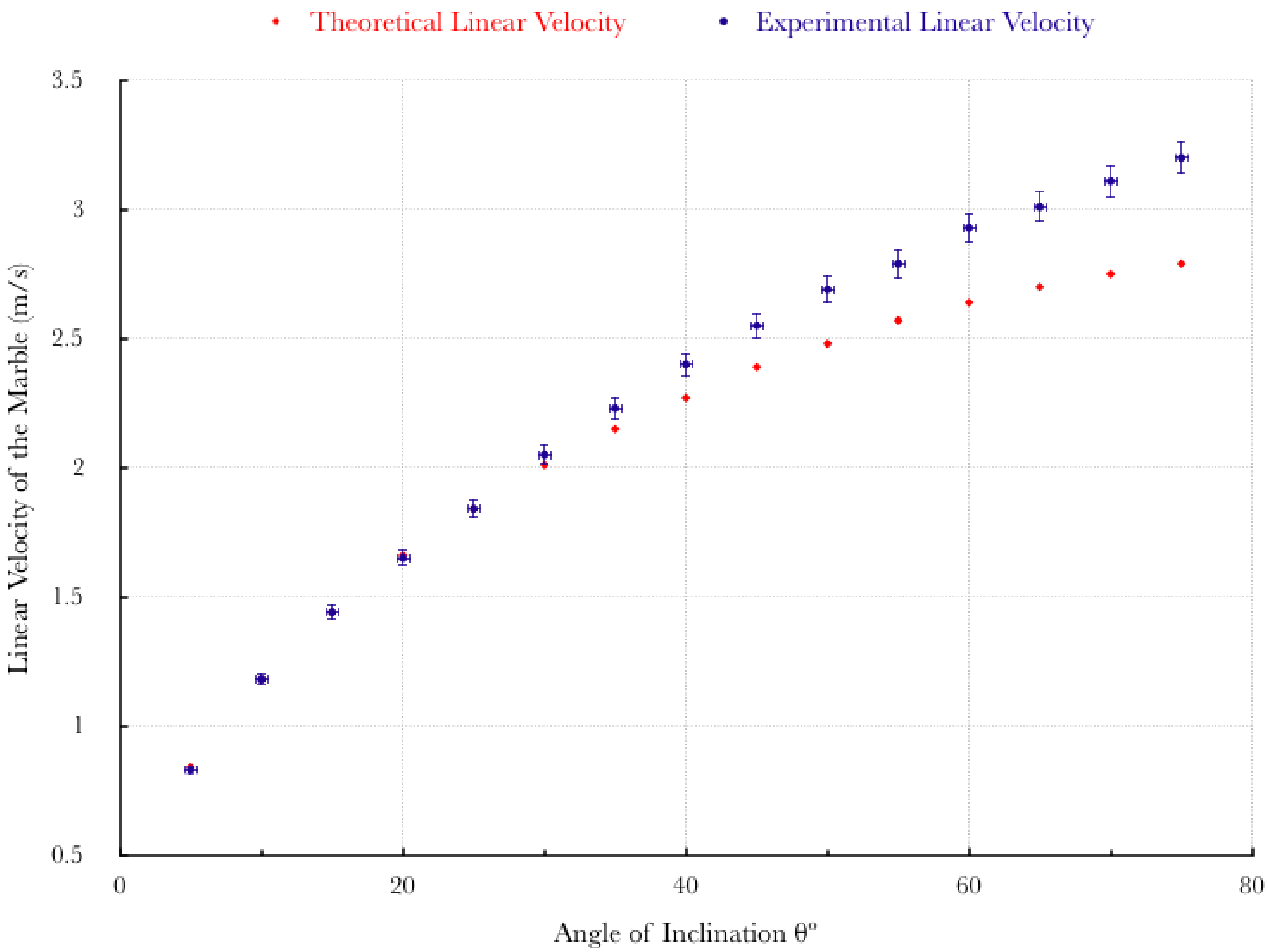

Figure 5: Theoretical vs experimental linear velocities at the bottom of the ramp for the marble

The data points from the angles 5º-25º align almost perfectly with theory. Though the data point corresponding to θ ∈ 30º is within experimental error, we notice that the trend for these values is to be greater than the theoretical velocity, and that these values start to diverge as the angle increase. At larger angles, the experimental velocities are outside the range of expected values, indicating that slippage has taken place because the gravitational potential energy was converted to more translational and less rotational kinetic energy due to the reducing frictional force, and slippage increases with increasing θ. A second observation is that the values for the two velocities flatten out toward larger values of θ, again supporting the square-root relationship of velocity vs θ.

An auxiliary consideration is what happens to the angular velocity of the marble as it crosses the photogate. Because it is incorrect to assume $v = r\omega$holds for larger angles, we cannot calculate the angular velocity by dividing the experimental velocity with the marble's radius. Using conservation laws, we arrive at the following relation for ω:

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$$\omega = \sqrt{\frac{10gdsin\,\theta - 5v^2}{2r^2}} \quad ......(13)(\text{Verma, 179})$$

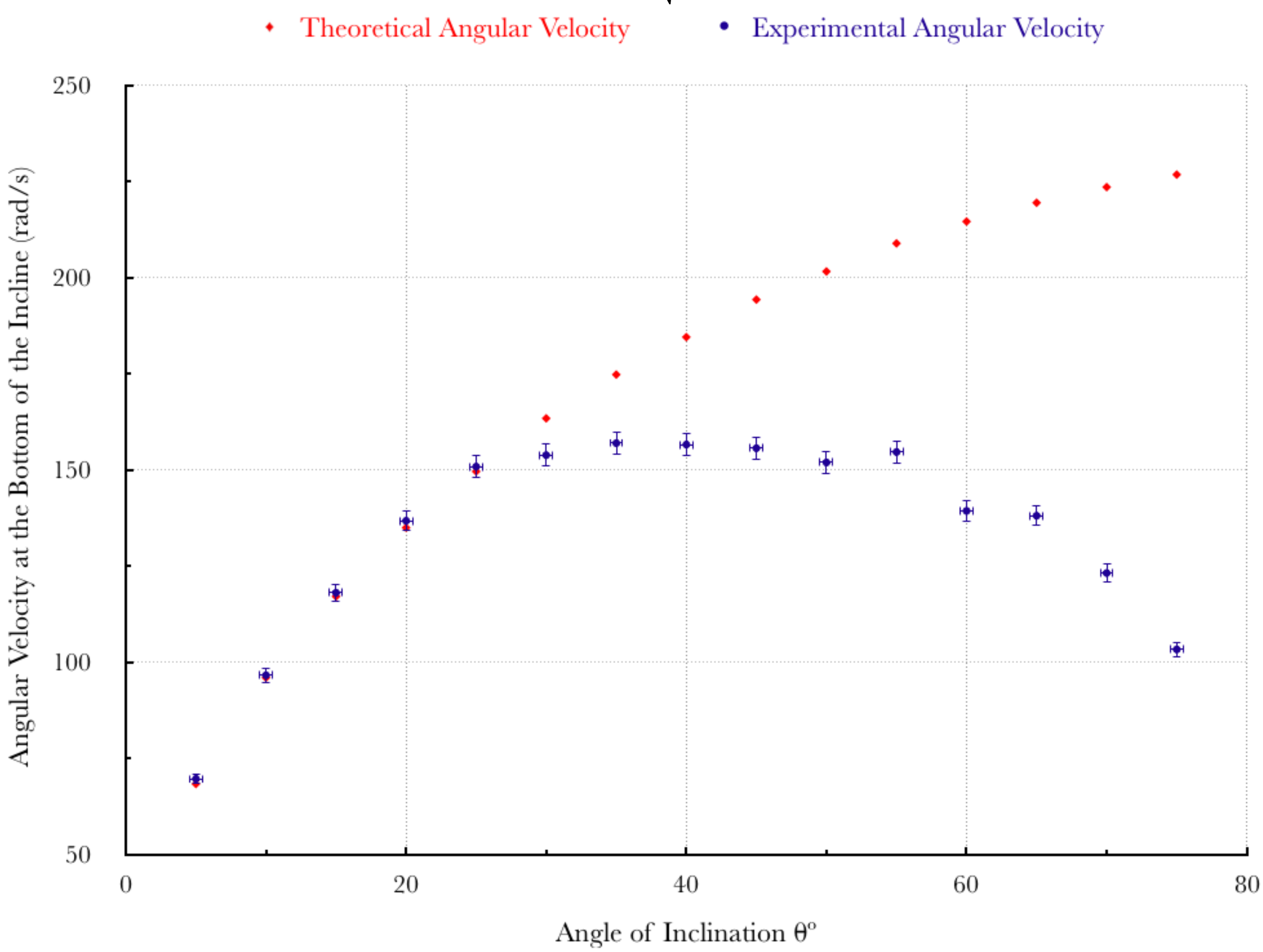

Figure 6: Theoretical vs experimental angular velocities at the bottom of the ramp for the marble

Substituting the relevant values of v in the above equation yields the graph shown above. We see from the graph that the individual data plots tend to be more scattered and random that the values for the theoretical velocity. Further, we notice that the angular velocity peaks around 35º and then dips down, indicating that the body begins to rotate less from this angle onward, indicating slippage is taking place.

Another important assertion we made was that no energy would be dissipated due to friction because the frictional force would generate the torque for rotation. To verify this, the values of the experimental linear and angular velocities were added and contrasted to the initial gravitational potential energy $mgdsin\theta$where d is the length of the incline, = (57.5 ± 0.1) cm; values were also extracted for the ratio E_t/E_r.

Table 3: comparison of linear and rotational kinetic energies for the marble

Angle of Incline θ (±0.5)°	Translatory Kinetic Energy (J)	Rotational Kinetic Energy (J)	E_t/E_r (± 4.31 %)	Total Experimental Energy (J)	Initial Potential Energy (J)
5	0.007	0.003	2.35	0.010	0.010
10	0.014	0.005	2.47	0.019	0.019
15	0.020	0.008	2.46	0.028	0.028
20	0.026	0.011	2.41	0.038	0.038
25	0.033	0.013	2.46	0.046	0.046
30	0.041	0.014	2.93	0.055	0.055
35	0.048	0.015	3.34	0.063	0.063
40	0.056	0.014	3.88	0.070	0.070
45	0.063	0.014	4.43	0.078	0.078
50	0.070	0.014	5.18	0.084	0.084
55	0.076	0.014	5.38	0.090	0.090
60	0.084	0.011	7.31	0.095	0.095
65	0.088	0.011	7.85	0.099	0.099
70	0.094	0.009	10.54	0.103	0.103
75	0.100	0.006	15.86	0.106	0.106

Similar to the trends with the experimental velocity, the ratio tends to (within error) 2.5, as predicted by theory. However, it increases rapidly with increasing θ, reaching 15.86 for 75° . The plausible explanation is that the body slips at higher angles, resulting in greater linear velocity and hence more translational kinetic energy. This, however, is speculation and the hypothesis that it is the lack of friction which is responsible for producing slippage, and will be dealt with later.

We notice how perfectly energy has been conserved before and after the experiment by referring to the rightmost two columns. This supports our theory of friction not being dissipative.

Finally, to understand how slippage has the opposite effect on the marble's experimental linear velocity as compared to its angular velocity, consider the plot of E_t/E_r

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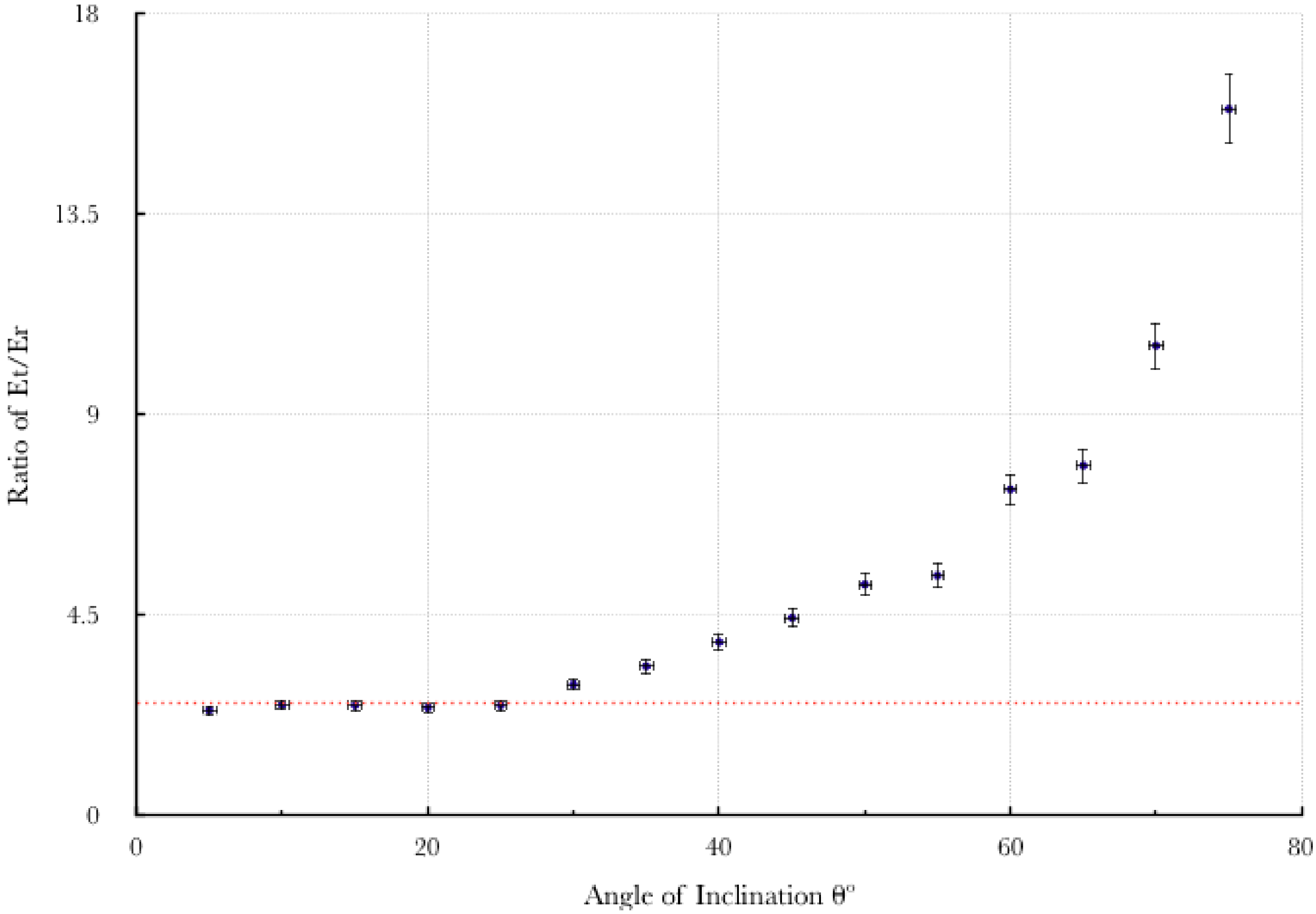

Figure 7: Ratio of E_t/E_r for the rubber ball

V. CONFIRMATION OF HYPOTHESIS USING A RUBBER BALL

Results have concluded that the marble does indeed slip after 30°, but to verify whether the slippage is caused by the lack of friction, we can perform another experiment where the coefficient of static friction is significantly higher, and observe whether slipping still occurs at that angle. A rubber ball of radius (2.68 ± 0.005) cm and mass (14.96 ± 0.01) g was chosen for this experiment, and by repeating the set-up on page 9 its coefficient of static friction with the plane was measured to be 0.969 ± 0.017, substantially higher than that of the marble.

Pythagoras' theorem was again used to measure the distance the beam covered, measured as (2.53 ± 0.03) cm. The % uncertainty in the experimental velocity is less than the same uncertainty for the marble, as the rubber ball's greater radius reduces the % uncertainty in measuring the distance the beam cuts through. The 8 trials were conducted again, and the velocity of the rubber ball measured:

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Table 4: experimental vs theoretical velocity of the rubber ball for a range of angles

Angle of Incline θ (±0.5)°	Mean Beam Cutting Time (± 10^{-5} s)	Experimental Velocity (ms^{-1})(±0.98%)	Theoretical Velocity (ms^{-1}) ≈(± 0 %)
5	0.0304	0.83	0.84
10	0.02127	1.19	1.18
15	0.01779	1.42	1.44
20	0.01534	1.65	1.66
25	0.01367	1.85	1.84
30	0.01229	2.06	2.01
35	0.01178	2.15	2.15
40	0.01098	2.31	2.27
45	0.0106	2.39	2.39
50	0.01026	2.47	2.48
55	0.00978	2.59	2.57
60	0.00943	2.68	2.64
65	0.00916	2.76	2.70
70	0.00892	2.84	2.75
75	0.00874	2.90	2.79

Two observations follow: first, compared to the marble, the experimental velocity, while still in range with the theoretical velocity upto 60°, aligns less perfectly. Timings for each trial [Appendix I] reveal a greater standard deviation than the corresponding values for the marble. A probable explanation is that a tiny rubber seal present on the ball's surface distorted its motion down the incline, causing greater inaccuracy in our measurements.

Second, and more importantly, the experimental velocity aligns with the theoretical velocity to a much larger angle than was the case with the marble.

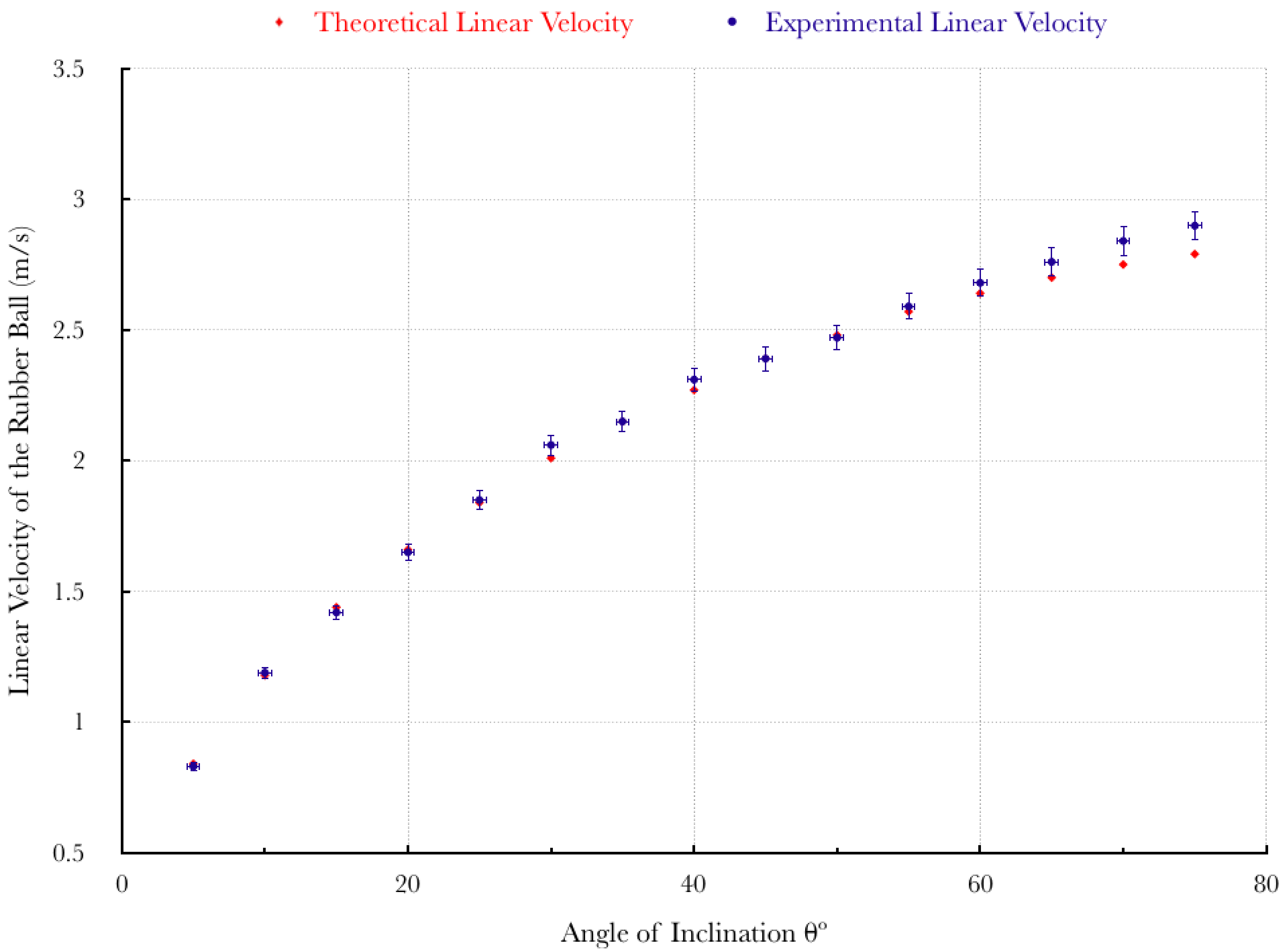

Figure 8: Theoretical vs experimental linear velocities at the bottom of the ramp for the rubber ball

The overlap of the experimental and theoretical values is far greater (upto 60º) for the rubber ball than the marble. This is because a greater frictional force develops between the rubber and the incline than with the marble, providing the torque needed for pure rolling at larger angles, clearly exemplifying that the greater the frictional force that develops between the body and the surface, the less slippage will occur, an observation in line with our hypothesis.

VI. EVALUATION OF RESULTS

The investigation has yielded several significant results.

We noticed that the marble began to slip at around 30º-35º, invalidating our initial prediction of (42.7 ± 0.3)º for θ_c. Still, our results were within reasonable range and a possible explanation for this disparity is that errors were committed while measuring the coefficient of static friction, causing us to overestimate the value of μ_s. We failed to account for the additional force of friction as the string passes over the pulley; in principle, we assume the string to be massless and hence frictionless, but this assumption breaks down in reality. This caused us to add a greater quantity of salt to the cup, overestimating the true mass needed to cause the marble to just slide had the string been truly frictionless.

We can see that during slippage, the linear velocity increases quite rapidly as we increase θ, while the angular velocity peaks at around 35^{o} and then decreases, supporting our theory of slippage causing greater translation and less rotation.

A criticism of our experimental set-up is that the $\omega$was measured indirectly using conservation laws instead of experimentally, which doubled the uncertainty for the E_t/E_r ratio. Had we set up a Vernier Rotary Motion Sensor (which was unfortunately not part of our school laboratory), we would have been able to analyse the graph of angular velocity versus θ directly using primary data.

For pure rolling, we expect E_t/E_r to be 2.5 independent of the angle of inclination. Hence, the fact that the ratio climbs so rapidly from 7.85 for 65^{o} to 15.86 for 75^{o} demonstrates how rapidly slippage increases at larger angles.

our observation that slipping was far less prominent with the rubber ball proves our hypothesis that it is the reduced frictional torque that causes the centre of mass of the marble to accelerate faster down the incline, and rotate less. Indeed, given $\theta > 35^{o}$, the relation $v = r\omega$does not hold true. However, even with the rubber ball, slippage occured at an angle less than $\approx 73.5^{o}$ (θ_c for the rubber ball provided $\mu_s = 0.969 \pm 0.017$, supporting our evaluation of the string producing another source of friction).

Several sources of error were apparent in our investigation: firstly, the axis of the ball's rotation might not have been exactly parallel to the incline, given that we could observe the ball being displaced horizontally from its point of release as it crossed the photogate. Further, we assume that the marble had a uniform mass distribution; this may have not been the case because of the 'swirl' (amorphous, non-uniform, coloured shards of glass) inside the marble [view figure 3], causing the marble's moment of inertia to not adhere perfectly to its theoretical value.

Another consideration is that the incline plane's surface might not have been completely smooth. While the acrylic was cleaned with a dry cloth before the investigation, any scratches present on the surface would have slowed down the marble. Moreover, a surface with scratches would have yet again introduced a source of unreliability for our value of μ_s, because it would give an anisotropic magnitude of friction.

There are two primary improvements which might have increased the strength of our conclusion: firstly, it would have been prudent to set-up a motion sensor on the top of the incline, not to measure the marble's velocity but rather to track the position of the ball down the ramp as a function of time, to investigate whether slipping produced any abnormal trends in the above relationship - we presume that the position vs time graph would show significantly greater acceleration as the marble moves down the ramp for large values of θ.

Secondly, it would appear to be more scientific if instead of using a rubber ball for greater friction, we taped sandpaper to the acrylic, which itself would have increased the coefficient of static friction . This would enable us to keep the marble as a controlled variable, instead of having to calculate a new value for the distance the light beam crosses with the rubber ball.

VII. CONCLUSION

Despite the aforementioned limitations, the investigation can be deemed a success because of the extreme accuracy in our processed data as compared to literature, and the conclusion it supports. The investigation clearly demonstrates how the angle of incline affects the slippage produced in a marble down an incline. The marble's experimental linear velocity aligns perfectly with literature upto an angle θ_c, after which slippage occurs, and the difference between the experimental and literature values for the linear velocities increases as we increase θ for $\theta > \theta_c$. our value of $\theta_c = 30^{o}$-35^{o} is only a rough estimate of the value predicted by theory (42.7^{o}).

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This investigation has several practical applications: understanding slippage in machine parts or the tyres of a car has the potential to minimize power consumption. We could perhaps model the slippage of the wheels of an aircraft, on perform this experiment with bodies of different shapes: cylinders, rings, etc. or even try to model the motion of a cylinder with holes on its surface, or one filled with fluids of varying densities - the investigation's scope is limitless.

VIII. ACKNOWLEDGEMENTS

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Programmes Used:

1. *Photogate, © 2018 Vernier Software & Technology, LLC.*
2. *LabQuest 2, © 2018 Vernier Software & Technology, LLC.*
3. *Microsoft office Suite, © 2018 Microsoft .*

APPENDICES

Appendix I

Raw data used in determining the coefficient of static friction μ_s between the incline and rubber ball:

Table 5: Coefficient of static friction between the incline plane and the rubber ball

Trial No.	Mass of Cup M in g (± 0.01 g)	Coefficient of Static Friction μ_s
1	14.63	0.977
2	14.72	0.983
3	14.61	0.975
4	14.22	0.949

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5	14.56	0.972
6	14.34	0.957
Mean	**14.51 ± 0.25**	**0.969 ± 0.017**

Appendix II

5 Degrees:

Table 6: Mean beam cutting time values of the marble and rubber ball for 5º

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.02876	0.03009
Trial 2	0.02833	0.03046
Trial 3	0.02809	0.02987
Trial 4	0.02846	0.03098
Trial 5	0.02863	0.03087
Trial 6	0.02850	0.03031
Trial 7	0.02849	0.03017
Trial 8	0.02858	0.03047
Trial Average	**0.02848**	**0.03040**

10 Degrees:

Table 7: Mean beam cutting time values of the marble and rubber ball for 10º

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.01996	0.02220
Trial 2	0.02026	0.02118
Trial 3	0.02001	0.02122
Trial 4	0.02011	0.02101
Trial 5	0.01992	0.02118
Trial 6	0.02029	0.02098
Trial 7	0.02013	0.02103
Trial 8	0.01990	0.02139
Trial Average	**0.02007**	0.02127

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15 Degrees:

Table 8: Mean beam cutting time values of the marble and rubber ball for 15º

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.01638	0.01765
Trial 2	0.01646	0.01744
Trial 3	0.01663	0.01799
Trial 4	0.01644	0.01809
Trial 5	0.01638	0.01776
Trial 6	0.01628	0.01789
Trial 7	0.01643	0.01766
Trial 8	0.01639	0.01782
Trial Average	**0.01642**	**0.01779**

20 Degrees:

Table 9: Mean beam cutting time values of the marble and rubber ball for 20º

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.01439	0.01544
Trial 2	0.01429	0.01567
Trial 3	0.01435	0.01522
Trial 4	0.01427	0.01543
Trial 5	0.01462	0.01499
Trial 6	0.01430	0.01518
Trial 7	0.01425	0.01540
Trial 8	0.01446	0.01539
Trial Average	**0.01437**	**0.01534**

25 Degrees:

Table 10: Mean beam cutting time values of the marble and rubber ball for 25º

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.01283	0.01345
Trial 2	0.01291	0.01383

Trial 3	0.01289	0.01356
Trial 4	0.01288	0.01378
Trial 5	0.01285	0.01365
Trial 6	0.01289	0.01370
Trial 7	0.01290	0.01392
Trial 8	0.01293	0.01349
Trial Average	**0.01289**	**0.01367**

30 Degrees:

Table 11: Mean beam cutting time values of the marble and rubber ball for 30°

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.01163	0.01232
Trial 2	0.01152	0.01226
Trial 3	0.01151	0.01219
Trial 4	0.01161	0.01245
Trial 5	0.01159	0.01197
Trial 6	0.01143	0.01236
Trial 7	0.01147	0.01243
Trial 8	0.01152	0.01232
Trial Average	**0.01154**	**0.01229**

35 Degrees:

Table 12: Mean beam cutting time values of the marble and rubber ball for 35°

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.01066	0.01187
Trial 2	0.01075	0.01160
Trial 3	0.01080	0.01198
Trial 4	0.01059	0.01175
Trial 5	0.01054	0.01165
Trial 6	0.01055	0.01200

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Trial 7	0.01042	0.01168
Trial 8	0.01058	0.01171
Trial Average	**0.01061**	**0.01178**

40 Degrees:

Table 13: Mean beam cutting time values of the marble and rubber ball for 40°

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.00981	0.01091
Trial 2	0.00984	0.01110
Trial 3	0.00993	0.01103
Trial 4	0.00992	0.01072
Trial 5	0.00984	0.01087
Trial 6	0.00977	0.01099
Trial 7	0.00989	0.01112
Trial 8	0.00983	0.01109
Trial Average	**0.00985**	**0.01098**

45 Degrees:

Table 14: Mean beam cutting time values of the marble and rubber ball for 45°

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.00927	0.01073
Trial 2	0.00927	0.01056
Trial 3	0.00929	0.01043
Trial 4	0.00923	0.01058
Trial 5	0.00928	0.01069
Trial 6	0.00931	0.01091
Trial 7	0.00926	0.01036
Trial 8	0.00930	0.01055
Trial Average	**0.00928**	**0.01060**

50 Degrees:

Table 15: Mean beam cutting time values of the marble and rubber ball for 50°

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.00874	0.01016
Trial 2	0.00881	0.01043
Trial 3	0.00879	0.01046
Trial 4	0.00886	0.01002
Trial 5	0.00883	0.00998
Trial 6	0.00890	0.01032
Trial 7	0.00883	0.01042
Trial 8	0.00879	0.01025
Trial Average	**0.00882**	**0.01026**

55 Degrees:

Table 16: Mean beam cutting time values of the marble and rubber ball for 55°

Trial No.	Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)	Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)
Trial 1	0.00841	0.00988
Trial 2	0.00840	0.00956
Trial 3	0.00850	0.00947
Trial 4	0.00854	0.01008
Trial 5	0.00851	0.00989
Trial 6	0.00852	0.01003
Trial 7	0.00849	0.00941
Trial 8	0.00848	0.00995
Trial Average	**0.00848**	**0.00978**

60 Degrees:

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Table 17: Mean beam cutting time values of the marble and rubber ball for 60°

Trial No.	**Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)**	**Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)**
Trial 1	0.00814	0.00965
Trial 2	0.00816	0.00923
Trial 3	0.00815	0.00928
Trial 4	0.00812	0.00935
Trial 5	0.00809	0.00946
Trial 6	0.00813	0.00956
Trial 7	0.00811	0.00949
Trial 8	0.00812	0.00939
Trial Average	**0.00813**	**0.00943**

65 Degrees:

Table 18: Mean beam cutting time values of the marble and rubber ball for 65°

Trial No.	**Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)**	**Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)**
Trial 1	0.00783	0.00912
Trial 2	0.00782	0.00902
Trial 3	0.00795	0.00923
Trial 4	0.00786	0.00893
Trial 5	0.00792	0.00909
Trial 6	0.00785	0.00921
Trial 7	0.00784	0.00942
Trial 8	0.00788	0.00923
Trial Average	**0.00787**	**0.00916**

70 Degrees:

Table 19: Mean beam cutting time values of the marble and rubber ball for 70°

Trial No.	**Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)**	**Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)**
Trial 1	0.00750	0.00879
Trial 2	0.00765	0.00859
Trial 3	0.00760	0.00901

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Trial 4	0.00757	0.00897
Trial 5	0.00759	0.00889
Trial 6	0.00770	0.00898
Trial 7	0.00769	0.00908
Trial 8	0.00775	0.00902
Trial Average	**0.00763**	**0.00892**

75 Degrees:

Table 20: Mean beam cutting time values of the marble and rubber ball for 75°

Trial No.	**Mean Beam Cutting Time for the Marble ($\pm 10^{-5}$ s)**	**Mean Beam Cutting Time for the Rubber Ball ($\pm 10^{-5}$ s)**
Trial 1	0.00742	0.00867
Trial 2	0.00739	0.00862
Trial 3	0.00739	0.00892
Trial 4	0.00741	0.00853
Trial 5	0.00741	0.00876
Trial 6	0.00743	0.00883
Trial 7	0.00742	0.00887
Trial 8	0.00743	0.00869
Trial Average	**0.00741**	**0.00874**